

# The Development of Proportional Reasoning: Effect of Continuous Versus Discrete Quantities

Yoonkyung Jeong  
*Catholic University of Korea*

Susan C. Levine  
*University of Chicago*

Janelle Huttenlocher  
*University of Chicago*

This study examines the development of children's ability to reason about proportions that involve either discrete entities or continuous amounts. Six-, 8- and 10-year olds were presented with a proportional reasoning task in the context of a game involving probability. Although all age groups failed when proportions involved discrete quantities, even the youngest age group showed some success when proportions involved continuous quantities. These findings indicate that quantity type strongly affects children's ability to make judgments of proportion. Children's greater success in judging proportions involving continuous quantities appears to be related to their use of different strategies in the presence of countable versus noncountable entities. In two discrete conditions, children—particularly 8- and 10-year-olds—adopted an erroneous counting strategy, considering the number of target elements but not the relation between target and nontarget elements, either in terms of number or amount. In contrast, in the continuous condition, when it was not possible to count, children may have relied on an early developing ability to code the relative amounts of target and nontarget regions.

Studies of children's ability to reason about proportions have yielded a mixed and somewhat confusing pattern of results. Some studies show competence on tasks

involving proportional reasoning as early as the preschool years. Others, beginning with the classic studies carried out by Piaget and Inhelder (Inhelder & Piaget, 1958, Piaget & Inhelder, 1975) have indicated that proportional reasoning is a late emerging skill, not developing until about age 11. In fact, Piaget and Inhelder argued that the ability to compare proportions involves reasoning about the “relation between relations,” a hallmark of formal operational thinking. We currently lack an understanding of what accounts for this discrepancy in findings. In this study, we examine a hypothesis suggested by the pattern of existing findings—in particular, that children are able to reason about proportions involving continuous amounts long before they are able to do so for sets of discrete entities. We suggest that young children rely on an early emerging ability to perceptually code relative amount in reasoning about the proportional relation of continuous quantities. In contrast, in the context of discrete entities, young children may instead use erroneous counting strategies and ignore the perceptual relation of the relevant quantities. To examine our hypothesis, we compare performance on parallel proportion problems that involve either discrete or continuous quantities. Like Piaget and Inhelder, we examine children’s proportional reasoning in the context of a probability judgment task.

On a typical Piagetian proportional reasoning problem, children are presented with two sets of red and white marbles that differ in both the absolute number of red and white marbles and the proportion of red and white marbles (Piaget & Inhelder, 1975). The child’s task is to choose the set that is more likely to yield a red marble on a random draw. Until age 11, children tend to select the set with the greater absolute number of red (target) marbles rather than the set with the higher proportion of red marbles. A subsequent study by Noelting (1980) also showed that young children have difficulty relating two relative quantities. Noelting presented 6- to 16-year-old children with two sets of cups. In each set, some cups were filled with orange concentrate and others were filled with water. Children were asked to decide which set of cups would yield a stronger orange taste when mixed. Similar to Piaget and Inhelder’s marble studies, children did not succeed until they were 12 years of age, typically responding on the basis of the number of cups filled with orange concentrate without considering the number of cups filled with water. Consistent with these findings, many other researchers have provided evidence that proportional reasoning develops quite late (Chapman, 1975; Hoemann & Ross, 1971; Karplus, Pulos, & Stage, 1983; Noelting, 1980; Siegler & Vago, 1978). Children’s difficulty in all of these studies was related to focusing on only one aspect of the quantities (e.g., number of red marbles), rather than coding the relation between quantities (e.g., the number of red marbles in relation to the number of white marbles [part–part reasoning] or the number of red marbles in relation to the total number of marbles [part–whole reasoning]). Thus, difficulty in reasoning about the “relation between relations” might be attributable to difficulty in coding each of the relevant relations (e.g., Sophian & Wood, 1997).

In stark contrast to these findings, many studies have reported that preschool children can reason about proportions (Acredolo, O'Connor, Bank, & Horobin, 1989; Davies, 1965; Goswami, 1989; Hoemann & Ross, 1971; Huttenlocher, Newcombe & Vasilyeva, 1999; Lovett & Singer, 1991; Mix, Levine & Huttenlocher, 1999; Singer-Freeman & Goswami, 2001; Sophian & Crosby, 1999; Spinillo and Bryant, 1991; Yost, Siegel & McMichael, 1962). Several of these studies have used an information integration approach and found that young children can reason in situations that appear to call for proportional reasoning (Acredolo et al., 1989; Schlottmann, 2001). In these experiments, children were asked to judge the likelihood of an event outcome using a continuous rating scale. For example, in a study by Acredolo et al. (1989), children estimated the likelihood of a bug landing on a flower in a box that contained both flowers and spiders, using a continuous rating scale (e.g., happy face scale). As early as 5 years of age, children's estimates reflected the proportional relation between the quantity of bugs and flowers. In another study using this approach, Schlottmann (2001) found that children as young as 6 could successfully make judgments about expected value of complex gambles that involved integrating the probability and value of outcomes. These studies may have yielded different results than those found by Piaget and Inhelder (1975) because they tapped more implicit probabilistic notions.

Other studies have examined young children's proportional reasoning in the context of map-reading tasks. Huttenlocher et al. (1999), for example, found that young children can carry out proportional translation from one scale to another on a simple map-reading task. Three and 4-year-old children were given a picture of a rectangle that represented a long sandbox; a dot in the rectangle indicated where a toy was hidden in the sandbox, and children were asked to use this map to point to the actual location of the hidden object in the sandbox. The results showed that all 4-year-olds and more than half of the 3-year-olds in the study were able to translate distance along a single dimension in the context of this simple mapping task. In a subsequent study, Vasilyeva and Huttenlocher (2004) further showed that 5-year-old children could make a proportional judgment in a map that involved a two-dimensional spatial layout.

Using a different type of proportional matching task, Spinillo and Bryant (1991) presented 4- to 7-years-olds with a picture of a rectangle composed of blue and white regions and asked them to choose which one of two different block models matched the picture. This task required proportional reasoning because the picture and the block models were constructed on different scales. In addition, on some trials, the picture was presented in a different orientation than the block model. By 6 years of age, children had some success in matching the proportion shown in the picture to that shown in a block model. Similarly, Sophian and colleagues (Sophian, 2000; Sophian & Crosby, 1999; Sophian & Wood, 1997) also showed that preschool children were able to successfully match shapes that share proportional relations. By 5 years of age, they could make proportional matches on

the basis of height–width ratio of rectangles and also on the basis of the size ratios of figure segments, even when the segments in the target and choice figures were arranged somewhat differently. These findings contrast with those of Piaget and Inhelder (1967), who found that children did not match rectangles accurately until about 8 years of age, instead choosing matches that exaggerated the height–width relations of a target. Sophian (2000) suggested that this result may have been because the children in their study construed the task of finding the same shape in a more qualitative manner. Finally, using an analogical reasoning task, Goswami (1989) found that 4-year-old children could match proportional amounts of different geometric shapes. For example, if children were shown a picture where one-half of a circle matches one-half of a rectangle, then children could judge that one-fourth of a circle matches one-fourth of a rectangle.

Mix, Huttenlocher, and Levine (2002) pointed out that a salient difference between many studies reporting early versus later success on proportional reasoning tasks is the kind of quantity involved. In particular, tasks reporting early success involved relations between continuous quantities (e.g., Goswami, 1989; Huttenlocher et al., 1999; Lovett & Singer, 1991; Sophian, 2000; Spinillo & Bryant, 1991), whereas tasks reporting later success involved relations between discrete quantities (e.g., Noelting, 1980; Piaget & Inhelder, 1975). Mix et al. suggested that continuous quantities may facilitate proportional reasoning because such quantities are often seen in containers that can serve as a perceptual referent (e.g., the glass is half full). Children may code the relative portion of the container occupied (part–whole coding) or the relation between the occupied and empty part of the container (part–part coding). Supporting the possibility that containers facilitate proportional reasoning, Huttenlocher, Duffy, and Levine (2002) showed that 2- and 4-year-old children were able to match the size of a target dowel when it was presented in a container but not when it was presented alone. Indeed, even 6-month-old infants were able to code the length of a dowel when it was presented in a container (Gao, Levine, & Huttenlocher, 2000; Huttenlocher et al., 2002) but not when it was presented alone, without a perceptually salient referent (Huttenlocher et al., 2002). In a subsequent study, Duffy, Huttenlocher, and Levine (2005) showed that 4-year-olds are highly sensitive to the proportional relation between the extent of a dowel and a container. Participants were presented with a target dowel inside a container. They were then presented with two alternative dowels and asked to select the same dowel as the one shown in the original container (the one that belonged to a dog named Toby, a stuffed animal). Both alternative dowels were shown in the same size containers, but these containers differed in size from the original container. One of the choice dowels was the same absolute size as the original dowel and, thus, occupied a different proportion of the new container, and the other dowel was different from the original dowel in absolute size but occupied the same proportion of its container as the original dowel in the original container. Thus, the two alternatives differed as to whether they held constant absolute dowel

size or proportional dowel size. Consistent with Bryant's (1974) findings that relative coding emerges earlier than absolute coding, 4-year olds chose the dowel that had the same dowel–container relation as in the original display rather than the dowel that actually matched the target dowel in absolute size; by 8 years of age, children were able to base their judgment on absolute size. This early sensitivity to amount in relation to a physically present standard may underlie children's early success on proportional reasoning tasks involving continuous amounts.

Mix et al. (2002) also suggested several reasons why proportional reasoning may be difficult when the quantities involved are sets of discrete entities. First, although children may try to compare discrete sets using a strategy similar to that used with continuous quantities, this comparison may be difficult because the unit divisions between elements break up the perceptual gestalts of target and non-target regions. Second, when confronted with sets of discrete entities, children may engage in counting the target set rather than perceptually comparing target and non-target sets in terms of amount. Their reliance on counting in this situation may stem from the many experiences they have had counting the elements in sets. Even if children count both the target and non-target sets, the use of counting may be detrimental to children who cannot yet compute proportions in a numerically correct manner. The negative impact of counting on children's performance on proportional reasoning tasks is well exemplified by the kinds of errors described by Piaget and Inhelder (1975) on their marble task.

Singer-Freeman and Goswami (2001) carried out a study designed to compare directly children's ability to reason about proportions involving continuous and discrete quantities. In this study, children were asked to make judgments about proportional equivalence. On the continuous task, two same-sized pizzas were presented, one for the experimenter and the other for the child. The experimenter's pizza was divided into eight equal pieces and the child's pizza was divided into four equal pieces. The experimenter removed a portion of her pizza ( $2/8$ ,  $4/8$ , or  $6/8$ ) and asked the child to remove the same portion from his or her pizza. On the discrete task, the procedure was the same except that the experimenter had a set of eight chocolate candies and the child had a set of four chocolate candies. Children performed significantly better on the continuous task than on the discrete task. These tasks, however, may not have tapped children's proportional reasoning. On the continuous task, even though the child's pizza and the experimenter's pizza were divided into a different number of pieces (4 vs. 8), the absolute size of the two pizzas was the same. Similarly, on the discrete task, although the child and the experimenter had a different number of candies in their sets (4 vs. 8), the total amount of candy in each set was the same because the size of the experimenter's candies was half the size of the child's candies. Figure 1 presents a schematic view of Singer-Freeman and Goswami's proportion task. Thus, on both the discrete and continuous tasks children could have been matching the absolute amount of "stuff" the experimenter removed rather than the proportion of pizza or candy removed.

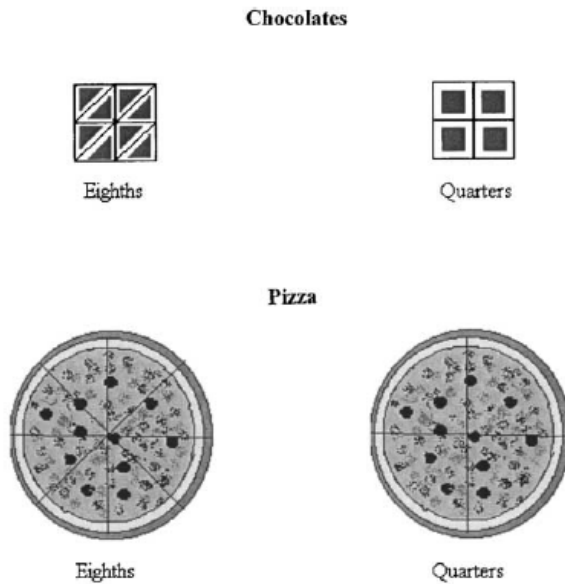


FIGURE 1 The schematic view of Discrete (chocolate candies) and continuous (pizza) items in Singer-Freeman and Goswami's (2001) study.

Such an absolute amount strategy may have been easier to apply in the context of a continuous pizza than in the context of discrete pieces of candy because amount is inherently continuous. Thus, Singer-Freeman and Goswami's finding of better performance in the continuous than the discrete condition does not provide conclusive evidence that proportional reasoning is easier in the context of continuous than discrete quantities.

Using an approach similar to that of Singer-Freeman and Goswami (2001), this study directly tests the hypothesis of an early advantage of continuous over discrete quantities in proportional reasoning. The task we used is a variation of Piaget and Inhelder's (1975) marble task in that children were asked to select a set that has a higher probability of yielding a target outcome. Children were presented with a donut-shaped spinning figure divided into red and blue regions. A black arrow was attached to the center of the donut so that it could point to either a blue or a red region when the donut stopped. Children were given a set of stickers and were told that if the arrow points to red when it stops spinning, they would get an additional sticker, but if it points to blue, they would lose one of their stickers. During test trials, children were presented with two donuts and were asked to select the one they would like to spin to get a sticker. Clearly, the

correct response was to choose the donut that had a larger proportion of red. To avoid the possibility of children basing their choices on absolute amount of the target region rather than on proportional amount, which was present in Singer-Freeman and Goswami's (2001) study, we varied the absolute size of the donuts being compared. We included a continuous condition and two different discrete conditions. In the continuous condition, target and non-target regions were both continuous and undivided. In this condition, it may be relatively easy for children to judge the proportional amount of target region in each donut by comparing the sizes of target and non-target regions in each. In the two discrete conditions, the target and non-target regions are divided into same-sized subregions (see Figure 2). The subdivided regions were adjacent to each other in the discrete adjacent condition and intermixed in the discrete mixed condition. Thus,

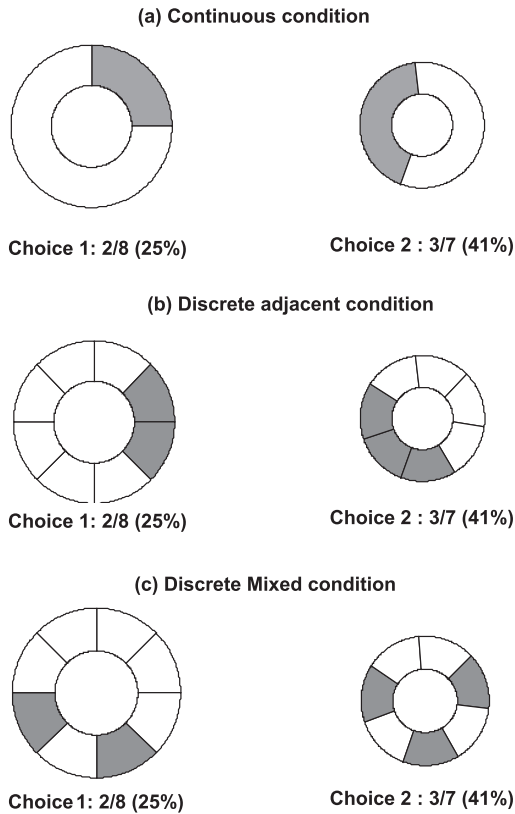


FIGURE 2 Examples of two alternative proportions in (a) continuous, (b) discrete adjacent and (c) discrete mixed conditions.

our discrete mixed condition is most similar to Piaget and Inhelder's (1975) task, in which red and white marbles were mixed in a container.

Three age groups, 6-, 8-, and 10-year-olds, were tested. We expected that the 6-year-olds might be able to perform at above chance levels in the continuous condition, given prior evidence of early sensitivity to proportional relations on tasks involving continuous quantities (e.g., Duffy et al., 2005; Huttenlocher et al., 1999; Spinillo & Bryant, 1991). In contrast, we expected that even 10-year-olds might struggle in the discrete conditions, given prior findings that school-age children have difficulty on proportional reasoning tasks involving discrete quantities (e.g., Noelting, 1980; Piaget & Inhelder, 1975). Participants were middle-class children in Seoul, Korea. As in the United States, Korean children are typically taught how to calculate proportions mathematically in fifth grade, when they are approximately 11 years old. In addition, 7- to 8-year-olds from Korea and the United States are both misled by the number of part in subsets when identifying labels for fractions, although Korean children perform at slightly higher levels on this task (Paik & Mix, 2003). Further, Korean children have been shown to outperform their U.S. peers in several cross-national studies of mathematical achievement (Miura, Okamoto, Vlahovic-Stetic, Kim & Han, 1999; Song & Ginsburg, 1987, 1988). Thus, it appears that children from both cultures make similar kinds of errors but that Korean children may on average develop their mathematical understandings somewhat more rapidly than children in the United States. In view of these findings, it seems likely that our results will generalize to the U.S. population, but perhaps be characteristic of slightly older children.

We designed our stimuli so that we would be able to examine the possibility that children's difficulty in coding proportion is related to their erroneous use of a counting strategy, which was accomplished by including both counting consistent and counting misleading discrete problems. In counting consistent problems, counting the number of target items in the two choices leads to a correct response whereas in counting misleading problems, counting the numbers of target items in the two choices leads to an incorrect response. If children erroneously compare the two alternative choices on the basis of the absolute number of target elements, their performance on counting consistent problems should be significantly above chance and their performance on counting misleading problems should be significantly below chance.

## METHOD

### Participants

Sixty children participated: 20 six-year-olds ( $M = 6.05$ ,  $SD = 0.32$ , range = 5.6–6.7; 12 boys and 8 girls), 20 eight-year-olds ( $M = 8.08$ ,  $SD = 0.37$ , range =

7.8–8.6; 8 boys and 12 girls), and 20 ten-year-olds ( $M = 10.04$ ,  $SD = 0.28$ , range = 9.8–10.5; 9 boys and 11 girls). The children were from middle-class families in the urban communities surrounding Seoul, Korea. Six-year-olds attended kindergarten, 8-year-olds attended second grade, and 10-year-olds attended fourth grade.

## Materials and Procedure

During the training phase, a donut-shaped spinning figure divided into red and blue regions was introduced to a child. The spinning figure was attached to the center of a 25 cm × 25 cm white-colored platform. A black arrow was attached to the center of the donut so that it could point to either a blue or a red region when the donut stopped. The spinning figure and the platform where it was attached were made of 5 mm-thick poster board. Children were given a set of stickers and were told that if the arrow points to red when it stops spinning, they would get an additional sticker, but if it points to blue, they would lose one of their stickers. To learn this rule, children played this game a few times with all three types of donuts (continuous, discrete adjacent, and discrete mixed) before beginning the test trials.

In the testing phase, each trial involved presenting two donut-shaped figures that differed in terms of overall size and in their proportion of red versus blue regions. The task was to select the donut with which they would like to play the spinning game to get more stickers. However, no spinning took place on test trials, and no stickers were given out during these trials to avoid the problem of differentially reinforcing children. Rather, each child was allowed to select three stickers at the end of the experiment. The overall sizes of these two alternative choices differed to prevent children from solving the problem by considering the absolute magnitude of the red region. For this purpose three different sized donuts were used, each with diameters of 12 cm, 15 cm, and 18 cm. These different sizes were chosen so that an absolute amount strategy would lead to chance level performance.

Trials were presented in three different conditions, as shown in Figure 2. In the continuous condition, the red and the blue regions were continuous and undivided. In the discrete adjacent condition, the colored regions were divided into same size subregions, and regions of the same color were adjacent to each other. In the discrete mixed condition, red and blue subregions were intermixed around the donut. Problems in each discrete condition were equally divided between “counting consistent” and “counting misleading” types, and the problems used in each condition were identical. Thus, the continuous condition included the same pairs of fractions used in the counting consistent and counting misleading conditions, although there actually was no such distinction in the continuous condition because counting is not an option. The problems used in each condition are summarized in Table 1.

Each participant was given 12 problems in each condition (continuous, discrete adjacent, and discrete mixed) in a blocked order, for a total of 36 trials. Three

TABLE 1  
The Structure of the Test Items

<i>Counting Type</i>	<i>Choice 1</i>		<i>Choice 2</i>		<i>Difference Between Two Choices</i>
	<i>Ratio</i>	<i>%</i>	<i>Ratio</i>	<i>%</i>	
Counting consistent	2/6	33	5/8	63	30
	4/10	40	5/8	62	22
	3/7	42	2/8	25	17
	3/5	60	5/7	71	11
	2/7	29	3/8	38	9
Counting misleading	2/7	28	1/5	20	8
	4/5	80	5/10	50	30
	3/5	60	4/10	40	20
	3/5	60	4/9	44	16
	5/7	71	6/10	60	11
	3/5	60	4/8	50	10
	4/6	67	6/10	60	7

*Note.* The above problems were applied to continuous, discrete adjacent and discrete mixed conditions.

different block orders were used, following a Latin square design. Problems within each block were presented in a fixed random order.

## RESULTS

### Performance Level in Each Quantity Type Condition: Comparison to Chance

Percentage correct scores are presented in Figure 3 for 6-, 8-, and 10-year-olds, respectively. Scores are shown for each quantity type condition (continuous, discrete adjacent, discrete mixed), for both counting consistent and counting misleading problems. Two tailed *t* tests were used to determine whether each age group's mean percentage correct in each condition differed from chance (50%). In the continuous condition, children in all age groups performed significantly above chance level: 6-year-olds,  $t(19) = 2.47, p \ll .05$ ; 8-year-olds,  $t(19) = 3.58, p < .01$ ; 10-year-olds,  $t(19) = 4.53, p < .01$ . Using the binomial distribution, we examined whether individual performance was consistent with the patterns observed using average scores. The number of children who were correct on at least 9 of 12 trials ( $p < .06$ ) increased with age (10, 15 and 19 out of 20 children for 6-, 8- and 10-year-olds, respectively).

In the discrete conditions, children's performance differed across age and for problems that were counting consistent versus counting misleading. Six-year-olds'

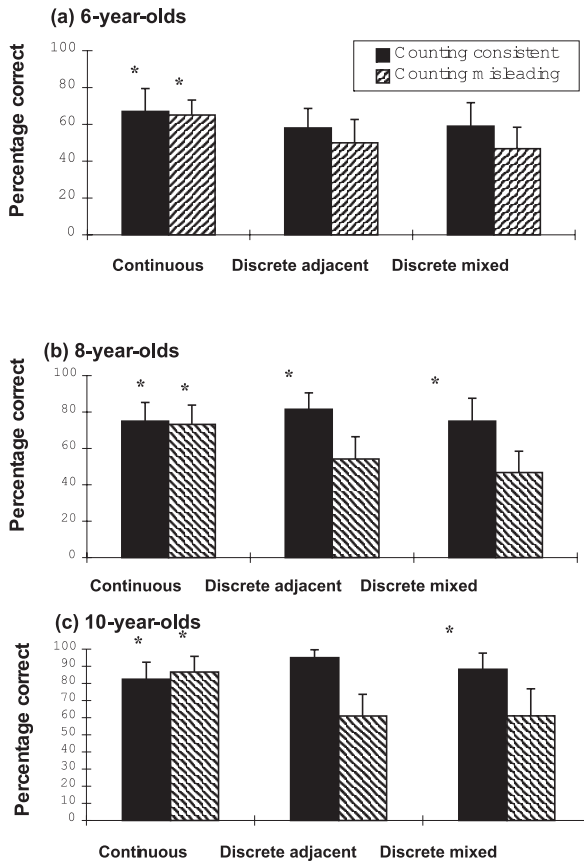


FIGURE 3 Percentage correct for different types of quantity in counting consistent and misleading problems for (a) 6-year olds, (b) 8-year-olds, and (c) 10-year-olds.

performance was not significantly above chance on either counting consistent problems, discrete adjacent,  $t(19) = 1.75, p = .095$ ; discrete mixed,  $t(19) = 1.59, p = .13$ , or counting misleading problems, discrete adjacent,  $t(19) = 0.00, p = 1.00$ , discrete mixed,  $t(19) = -0.63, p = .54$ . In contrast, 8- and 10-year-old children performed significantly above chance on both discrete adjacent and mixed problems that were counting consistent: 8-year-olds discrete adjacent,  $t(19) = 7.90, p < .01$ ; 8-year-olds discrete mixed,  $t(19) = 4.47, p < .01$ ; 10-year-olds discrete adjacent,  $t(19) = 21.03, p < .01$ ; 10-year-olds discrete mixed:  $t(19) = 9.10, p < .01$ . In contrast, their performance did not differ from chance on either discrete adjacent or mixed problems that were counting misleading (8-year-olds discrete adjacent,  $t(19) = .78, p = .44$ ; 8-year-olds discrete mixed:  $t(19) = 1.37, p = .19$ ; 10-year-olds

TABLE 2  
 Number (Proportion) of Children Passing the Criterion by Counting Type  
 and Age Group on Discrete Conditions

Age	Counting Consistent		Counting Misleading	
	<i>n</i>	%	<i>n</i>	%
Discrete Adjacent				
6 years (n= 20)	3	15	1	5
8 years (n= 20)	6	30	0	0
10 years (n=20)	14	70	3	15
Discrete Mixed				
6 years (n= 20)	2	10	1	5
8 years (n= 20)	6	30	0	0
10 years (n=20)	13	65	6	30

Note. Criterion = 100% (6 out of 6 trials,  $p < .05$ ).

discrete adjacent,  $t(19) = 1.91, p = .07$ ; 10-year-olds discrete mixed:  $t(19) = 1.58, p = .19$ ). These results suggest that 8- and 10-year-old children erroneously based their judgments on the number of target elements in both discrete conditions. Although 6-year-olds did not perform significantly above chance on either counting consistent or misleading problems, they showed a trend in the same direction as the older children. Using the binomial distribution, we examined how many children performed above chance (6/6 trials,  $p < .05$ ) on counting consistent and counting misleading problems in the two discrete conditions. Table 2 reports the number and the percentage of children who passed the criterion on each condition and these data are consistent with group findings.

### Analysis of Variance

An analysis of variance was conducted on the arcsine transforms of percentage correct scores, with age group as a between-subjects variable and type of quantity (continuous, discrete adjacent, discrete mixed) and counting type (counting consistent, counting misleading) as within-subjects variables. There was a significant main effect of age,  $F(2, 57) = 16.28, p < .01$ . Pairwise comparisons (Scheffe  $S, p < .025$ , to control for multiple comparisons) reveals that performances of each adjacent age group significantly differed,  $p < .01$  in each case, reflecting an improvement in children's performance with increasing age (6-year-olds:  $M = 57.5\%$ ,  $SD = 11.4$ ; 8-year-olds:  $M = 67.5\%$ ,  $SD = 11.0$ ; 10-year-olds:  $M = 78.8\%$ ,  $SD = 10.3$ ). There was also a main effect of quantity type,  $F(2, 114) = 7.48, p < .001$ . Pairwise comparisons (Scheffe  $S, p < .025$  to control for multiple comparisons) shows significantly better performance in the continuous condition ( $M = 75.2\%$ ,  $SD = 4.04$ ) than in either of the discrete conditions, which did not differ significantly from

each other (discrete adjacent condition:  $M = 66.2\%$ ,  $SD = 3.73$ ; discrete mixed condition:  $M = 64.0\%$ ,  $SD = 3.56$ ). A main effect of counting type,  $F(1, 57) = 32.38$ ,  $p < .001$ , reflected children's better performance on counting consistent ( $M = 77.61\%$ ,  $SD = 3.61$ ) than on counting misleading problems ( $M = 61.36\%$ ,  $SD = 3.23$ ).

The analysis of variance also revealed a significant Age  $\times$  Quantity Type  $\times$  Counting Type interaction,  $F(4, 114) = 2.47$ ,  $p < .05$ . Paired comparison tests (Scheffe  $S$ ,  $p < .006$  to control for multiple comparisons) shows that this interaction reflects better performance by 8- and 10-year olds on counting consistent than on counting misleading problems in the discrete conditions ( $p < .006$  in both cases) but not in the continuous condition (8-year-olds,  $p = .713$ ; 10-year-olds,  $p = .386$ ), but no difference of this kind is found for the 6-year olds in any quantity type condition (discrete adjacent,  $p = .226$ ; discrete mixed,  $p = .138$ ; continuous:  $p = .584$ ). In summary, these results indicate that 8- and 10- year-olds may have based their choices in the discrete conditions on counts of the number of target items.

#### Additional Analyses on Counting Consistent and Counting Misleading Problems

We carried out additional analyses to examine whether 8- and 10-year-old children's pattern of responding in the discrete conditions was consistent with counting the number of target elements. If these children used a counting strategy whenever countable entities were present, their performance on counting misleading problems should have been significantly below chance. However, this was not the case. Rather, their performance on counting misleading problems was slightly above chance (See Figure 3). We next examined whether 8- and 10-year-old children counted the number of target items when the proportional difference between two choices was small, but not when it was large. Proportional differences between the two choices on each problem ranged from 7% to 30% (See Table 1), and performance level was significantly correlated with the magnitude of this proportional differences for all age groups in each of the three quantity conditions (in each case,  $r > .2$ ,  $p < .05$ ; see Table 3). These correlations indicate that children's performance level increased as the proportional difference between the two choices increased.

We next calculated a "counting index" for each proportional difference. This index was calculated by subtracting the percentage correct score each child received on counting misleading problems from the percentage correct score he or she received on counting consistent problems. A large difference in performance between counting consistent and counting misleading problems was considered to be indicative of the use of a counting strategy. If children were more likely to use a counting strategy when proportional differences between two choices were small, making it difficult to judge the proportional difference perceptually, we would obtain a significant negative correlation between the counting index and the propor-

TABLE 3  
R Values for Correlations Between Proportional Differences  
and Percentage Correct for Each Quantity Condition for Each Age Group

Age	Quantity type		
	Continuous	Discrete adjacent	Discrete mixed
6 year	0.371**	0.259**	0.233**
8 year	0.303**	0.298**	0.345**
10 year	0.218*	0.180*	0.243**

\*  $p < .05$  . \*\*  $p < .01$ .

TABLE 4  
R Values for Correlations Between Proportional Differences  
of the Choices and Counting Index

Age	Quantity type	
	Discrete Adjacent	Discrete Mixed
6 year	-0.038	-0.053
8 year	-0.298**	-0.355 **
10 year	-0.183*	-0.222**

\*  $p < .05$  . \*\*  $p < .01$ .

tional difference between the two choices. For 8- and 10-year olds, significant negative correlations, stronger for 8- than 10-year-olds, were found in both discrete conditions (see Table 4) whereas these correlations were not significant for 6-year-olds in any quantity condition. Thus, the smaller the proportional differences between the two alternative choices, the more frequently a counting strategy appears to have been used by the 8- and 10-year-olds. When the proportional difference between the two choices on a problem was large, 8- and 10-year-old children may have attempted to respond on the basis of perceived differences between the proportions of the target versus target areas, similar to the way they responded in the continuous condition. In contrast, when the proportional difference between the two choices on a problem was small, these children, particularly 8-year-olds, appear to have counted the number of target regions, leading to correct responses on counting consistent problems and incorrect responses on counting misleading problems.

Children's performance was compared with chance level (50%) using paired  $t$  tests, separately for problems with larger proportional difference (17–30%) and problems with smaller proportional differences (7–11%). Consistent with our assumption, when proportional differences were large, performance of 8- and

10-year olds was significantly above chance in both discrete conditions, even on counting misleading problems: 8-year-olds discrete adjacent,  $t(19) = 2.34, p < .05$ ; 8-year-olds discrete mixed,  $t(19) = 5.10, p < .01$ ; 10-year-olds discrete adjacent,  $t(19) = 2.94, p < .01$ ; 10-year-olds discrete mixed,  $t(19) = 3.87, p < .01$ . In contrast, when the proportional differences were small (7–11%), performance of the 8- and 10-year olds was significantly above chance only on counting consistent problems: 8-year-olds: discrete adjacent,  $t(19) = 6.20, p < .01$ , discrete mixed,  $t(19) = 4.15, p < .01$ ; 10-year-olds: discrete adjacent,  $t(19) = 10.19, p < .01$ , discrete mixed,  $t(19) = 8.79, p < .01$ . Eight-year-olds performed significantly below chance on counting misleading problems with small proportional in the discrete mixed condition,  $t(19) = -2.47, p < .05$ , and marginally below chance,  $t(19) = -2.01, p < .10$ , in the discrete adjacent condition, suggesting that they counted only the number of red regions. In this regard, the performance of the 8-year-olds differed from that of the 10-year-olds, whose performance on counting misleading problems with small proportional differences hovered around chance. A summary of each age group’s performance levels on the discrete problem types is presented in Table 5.

Finally, we examined the possibility that the ability to numerically compare two proportions is beginning to emerge in our oldest age group. Even though the average performance of 10-year-olds indicates that they erroneously counted the num-

TABLE 5  
Mean Proportion Correct (Se’s) on Each Discrete Condition According to Counting Types and Proportional Differences

Age	Discrete Adjacent			
	Small proportional difference		Large proportional difference	
	Counting consistent	Counting misleading	Counting consistent	Counting misleading
6 years	53.30 (31.42)	49.95 (28.76)	56.8 (26.85)	60.15 (22.43)
8 years	85.75**(22.87)	41.5(18.58)	81.7**(25.35)	68.35*(15.06)
10 years	91.7**(18.32)	53.3(27.32)	93.35**(7.39)	76.05**(30.46)
Age	Discrete Mixed			
	Small proportional difference		Large proportional difference	
	Counting consistent	Counting misleading	Counting consistent	Counting misleading
6 years	55 (33.61)	46.7 (22.82)	60 (31.24)	56.7 (29.71)
8 years	77.4**(28.82)	38.7*(-)(29.18)	77.3**(35.07)	73.7**(24.83)
10 years	88.4**(19.52)	56.6(31.69)	88.35**(22.35)	76.70**(30.87)

\*  $p < .05$ . \*\*  $p < .01$ . – performance was significantly lower than chance

ber of target items, some children in this age group did not adopt this strategy. We identified children who were correct on at least five out of six counting misleading problems ( $p < .108$ ) with a small proportional difference (three problems from the discrete adjacent condition and three problems from the discrete mixed condition), the kind of problem for which a perceptual strategy was least likely to succeed and for which counting the target items would lead to an incorrect answer. We found that 20% of 10-year-olds (4/20) but no 6- or 8- year-olds met this criterion. This result suggests that some 10-year-olds may be beginning to compare the two alternative proportions correctly in the discrete conditions, either in terms of relative amount, similar to the continuous condition, or in terms of numerical proportions.

## DISCUSSION

In this study, we examine children's ability to judge proportions in the context of a probability reasoning task. We specifically examine whether children are able to reason about proportions in the context of continuous amounts before they are able to do so in the context of discrete sets, and we find that this was the case. We find that all age groups tested (6-, 8- and 10-year-olds) performed better in the continuous condition than the discrete conditions. Further, all age groups performed at above chance levels in the continuous condition, whereas not even the 10-year-olds performed at above chance levels in the discrete conditions. Early success judging proportions involving continuous quantities may be related to the sensitivity to relative amount that is present in infants and young children (e.g., Bryant, 1974; Gao et al., 2000; Duffy et al., 2005; Huttenlocher et al., 2002). Our finding that children performed better on a proportional reasoning task involving continuous amounts than on a parallel task involving discrete quantities may help clarify a distinction made by Inhelder and Piaget (1958), almost a half century ago, when they wrote that children have a qualitative grasp of proportions before they are able to manipulate numerical proportions.

Children's better performance in the continuous than in the discrete conditions is likely related to the use of different strategies in these conditions. Although our results do not provide direct evidence about what kind of strategy children used in the continuous condition, it seems likely that they used a perceptual strategy similar to that used by infants and young children to code the extent of one length or region in relation to another (Duffy et al., 2005; Spinillo & Bryant, 1991). For example, children may have compared target and target areas in each donut (i.e., red versus blue areas [part-part coding] and then chosen the one that had proportionally more red area or have compared the red area in each donut to the total area of each donut [part-whole coding]). Alternatively, they may have compared the relative quantities perceptually by mentally magnifying the smaller donut until its size matched the size of the larger donut, similar to the processes used by young chil-

dren in map-reading tasks (Vasilyeva & Huttenlocher, 2004). It would then be possible to directly compare the absolute size of red regions in the two donuts. Such perceptual strategies do not result in an exact representation of proportion but do provide a systematic way to reason about proportion in an approximate manner.

Although it is also possible to use a perceptual strategy to solve problems in the discrete conditions, there are a number of reasons why children may not have done so. First, the lines that divide the regions into discrete pieces may break up the perceptual gestalt of target versus target areas, making it more difficult to compare areas. In the discrete mixed condition there is the added problem of accumulating intermixed target and non-target pieces into amounts that can be compared. The use of a magnification strategy, one of the possible perceptual strategies previously described, might also be more difficult with a number of small pieces than with two big regions. Second, the tendency to count the number of target pieces may have gotten in the way of using a perceptual strategy, especially for the 8- and 10-year-olds. This possibility is supported by the better performance of children in these age groups on counting consistent than counting misleading problems. However, our results also indicate that 8- and 10-year-olds do not apply a counting strategy indiscriminately. Rather, perceptual factors seem to have influenced their use of counting. That is, these children tended to count the number of target regions more frequently when the proportional difference between the red and blue regions on the two alternative spinners was small than when it was large. It is possible that with large proportional differences between the two alternatives, these children were more likely to treat the discrete problems like continuous problems, relying on a perceptual strategy.

Our results indicate that the reliance on an erroneous counting strategy may be diminishing by 10 years of age, which is evidenced by the somewhat smaller negative correlation between our counting index and the proportional difference between alternatives at age 10 than at age 8. In addition, a few of the 10-year-olds were beginning to succeed on these difficult problems. It is possible that they solved these problems using a perceptually based strategy, as they did with continuous quantities. Alternatively, these 10-year-olds may have learned how to calculate proportions mathematically, which would involve counting the number of target elements in relation to non-target elements or all elements in both choices, and then comparing parallel proportions. Testing older children and questioning them about what strategy they used would help differentiate these possibilities.

It is somewhat surprising that 6-year-olds did not use counting as consistently as the older groups in view of findings that young children frequently count when solving mathematical problems, even when this strategy is inappropriate (e.g., Miller, 1984). This result may have occurred because our task called for judgments about ordinal relations, which have been shown to be more difficult than judgments about equivalence relations (Mix et al., 2002). The demands of our ordinal judgment task may have led the 6-year-olds to try to use their early developing per-

ceptual strategy in lieu of counting, although they were unable to apply this strategy effectively in the discrete conditions. It is also possible that the 6-year-olds found it difficult to count the discrete elements because they were arranged in a circle with no separation except a thin black line rather than in a more typical aligned configuration with blank space between each element (e.g., Accredolo et al., 1989; Noelting, 1980).

Fuzzy trace theory (Brainerd & Reyna, 1990; Reyna & Brainerd, 1993) provides another possible explanatory framework for children's poor performance on the proportional reasoning problems involving discrete quantities. According to this theory, failure to extract relevant gist is a major source of errors in quantitative problem solving. For example, Reyna and Brainerd (1993) analyzed a news report stating that swings are the most dangerous type of playground equipment because the number of accidents involving swings is higher than the number involving other types of equipment. As they pointed out, this conclusion is erroneous because it fails to consider the base rate of how often children play on swings compared to other types of playground equipment, which would involve dividing the number of accidents by the frequency of usage. Similarly, the children in our study may have experienced difficulty on the proportional reasoning task in the discrete conditions because they only focused on the number of target elements and ignored the relevance of the number of target elements.

In summary, our findings demonstrate that children have an intuitive understanding of proportion in the context of continuous amounts long before they show this understanding in the context of discrete sets. We propose that their success on the continuous problems is based on the use of an early emerging ability to perceptually code and compare relative quantities (Bryant, 1974; Duffy et al., 2005; Spinillo & Bryant, 1991; Sophian, 2000; Sophian & Wood, 1997). In addition to increasing our understanding of the development of children's ability to reason about proportions, these findings have instructional implications. As was suggested by Resnick and Singer (1993, 1995), changes in instructional practice may be necessary for children to understand true ratio number in terms of their early intuitions about proportion. For example, drawing analogies between problems involving continuous and discrete quantities may help children overcome their tendency to apply incorrect numerical procedures to problems involving proportions. Such analogical strategies have been shown to be successful in enhancing children's learning in many domains (e.g., Gentner, Rattermann, Markman, & Kotovsky, 1995). Thus, children's understanding of proportions involving discrete sets may improve by having them perform a proportional reasoning task involving continuous quantities and then calling their attention to the similarity between this problem and a parallel problem involving discrete sets (e.g., drawing lines in to turn a continuous problem into a discrete one). This type of instruction may reduce the large developmental time lag between the emergence of children's intuitive notion of proportion in the context of continuous amounts versus discrete sets. In

turn, once children are able to map their intuitive notions of proportions on to discrete sets, they may be in a better position to understand the mathematical algorithms involved in calculating proportions.

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