Children's Understanding of Ruler Measurement and Units of Measure: A Training Study

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Abstract

An understanding of measurement and units of measurement is important in mathematics and science. However, children in the United States perform very poorly on measurement items on standardized mathematics tests compared to students in other countries. We examine understanding of linear measurement and units of measurement in 2nd grade students and carry out a training study aimed at improving their understanding, drawing on research showing that structural alignment is important in promoting learning. We find that students do not improve when training consists of measuring items aligned with a ruler, either when discrete units are coordinated with this activity by overlaying them on the ruler or when students engage in aligned ruler measurement and measurement with discrete units separately. However, we do find significant improvement when children measure objects that are misaligned and then aligned with the start of the ruler and discrete units are coordinated with ruler use.

Keywords: Mathematics development; cognitive development; structural alignment; linear measurement; units; ruler

Introduction

Measurement links the abstract world of number to the concrete world of objects, which have continuous properties such as linear extent, area, and volume. An understanding of measurement is important in mathematics and science achievement as well as in everyday life. Despite the importance of measurement, American children score lower on items assessing measurement knowledge than on items assessing knowledge of other mathematics topics. Children's difficulty understanding measurement is reflected in their persistent difficulty on NAEP linear measurement questions (86% errors in Grade 3; 78% Grade 4; 51% Grade 7; and 37% Grade 8; Carpenter et al., 1988; Lindquisit & Kouba, 1989). A part of their difficulty on such tests appears to be a lack of understanding of what a unit of measurement is.

Children are introduced to linear measurement early in elementary school. Central aspects of classroom instruction include practice measuring with conventional measurement instruments such as rulers and thermometers as well as practice measuring with nonstandard units such as paperclips. The question of whether conventional measurement instruments or discrete non-conventional units are more effective in teaching children about measurement is a matter of debate (e.g., Boulton-Lewis, 1996; Nunes, Light, & Mason, 1993). What is clear, however, is that children who are taught using the instructional methods in wide use in the United States, which make use of both of these types of measurement experience, are not gaining a deep understanding of measurement and measurement units. Rather, they seem to be gaining a set of procedural skills that make them appear to understand measurement.

The shallowness of children's understanding can be seen by examining children's performance on a simple task in which an object is misaligned with the "0"-point on a ruler (see Figure 1). Children typically make two types of errors on such problems (Lehrer, Jenkins, & Osama, 1998). One involves reading off the number on the ruler that aligns with the rightmost part of the object, i.e., responding "5 inches" in the case of the example in Figure 1. This type of error reflects reliance on a procedure that works perfectly as long as the object to-be-measured is aligned with the "0" point of a conventional ruler, but fails when the object is misaligned with this "0" point (Kamii, 2006; Martin & Strutchens, 2000). The other type of error involves counting the hash marks rather than the intervals or units that an object encompasses, i.e., responding "4 inches" for the crayon shown in Figure 1. This error type reflects an attempt to focus on units but a lack of understanding that the relevant

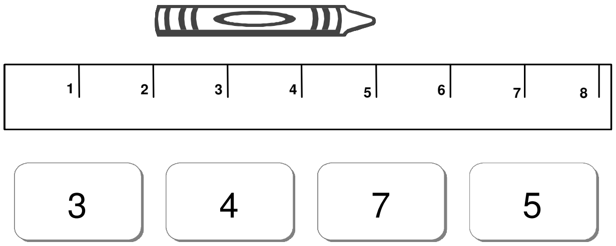

Figure 1: Example of a misaligned pre-test/post-test item.

units are inch-long intervals rather than hash marks. Again, this strategy works if the to-be-measured object is aligned with the ruler but not if it is misaligned. Children may acquire only a shallow understanding of linear measurement because of the kind of instruction they are receiving. That is, the measurement instruction they receive does not make use of structural alignment and comparison, which has been shown to be a powerful learning mechanism (Gentner, 1983; Gentner, Loewenstin & Hung, 2007; Loewenstein & Gentner, 2001). Under typical instructional conditions, children perform well when asked to measure with a ruler, merely reading off the number that aligns with the rightmost number on the ruler or counting hash marks, both of which will yield the correct answer if the object to be measured is aligned with the 0-point on the ruler. They also may perform well when measuring with discrete objects such as paper clips, meant to demonstrate the fundamental importance of units to measurement. However, because these measurement activities are not structurally aligned, the child may view them as separate and unrelated. Further, because they are not asked to measure the same objects when they are aligned and misaligned with the beginning of the ruler and to compare the answers obtained, they may fail to understand that the answers must be the same. Thus, because typical mathematics curricula do not invite comparisons between discrete units and the ruler nor comparisons between misaligned and aligned ruler measurements, children may fail to learn that ruler measurement is actually measurement with discrete units arrayed on a continuous instrument.

In the research reported below we examine the effectiveness of an instructional technique that is grounded in research showing that structural alignment is a powerful learning mechanism that allows children to map between different instances and arrive at deeper understandings (e.g., Gentner, 1983; Gentner et al., 2007; Loewenstein & Gentner, 2001). Our premise is that in the context of typical classroom instruction on measurement, children are not comparing activities that involve discrete units to activities that involve ruler measurement and have no opportunity to compare aligned and misaligned ruler measurement. Thus, when measuring with conventional instruments such as rulers they are employing procedural routines rather than conceptual knowledge of units. Because these procedures work well with the typically presented problems that involve aligning a ruler with a to-be-measured object, these procedures are reinforced. Moreover, because children's procedures typically produce the correct answer, teachers may believe that children have conceptual understanding of measurement. The cracks in their knowledge only become apparent when they are given novel problems such as the kind of misaligned ruler problem shown in Figure 1.

We predict that our experimental condition will allow the child to understand what linear units are and that ruler measurement involves counting these units. The instruction we use explicitly links measurement with discrete units to measurement with rulers. In addition, it shows the child that if the correct interval units are counted, the same measurement result is obtained whether the object being measured is aligned or misaligned with the ruler. Through structural alignment, the child is invited to compare ruler measurement with the discrete units that underlie it. This starkly contrasts with the situation that arises during typical classroom instruction, where the child is left to his/her own devices to draw analogies between ruler measurement and measurement with discrete units and has no experience with misaligned ruler problems.

The study reported below follows a pre-test, training, post-test design. Following the pre-test (Session 1), we taught children about units of measurement, and examined their learning by giving them an immediate post-test (Session 2). One week later we examined the retention of this learning by giving them a delayed post-test (Session 3). The training consists of two components that we hypothesize are important to increasing children's understanding. One of these components involved superimposing discrete one-inch units on a ruler. The superimposition of units on the ruler structurally aligned the ruler with the units (unlike the typical paper clip activity) and highlighted that linear units are intervals. After students responded on misaligned ruler items, typically incorrectly, we superimposed discrete units on top of the portion of the ruler that was aligned with the object being measured. After the child correctly counted the units to determine the length of the object, we moved the object to the "0" point on the ruler. This last step aligned the number of units obtained when units were counted when the object was misaligned with the ruler to the number of units obtained when the object was aligned with the "0" point of the ruler. We hypothesize that this step is critical in helping children conceptualize linear ruler measurement as a means of determining the number of units long an object is, rather than as a rote procedure of reading off the rightmost number on the ruler.

The effectiveness of this experimental training condition is assessed by comparing it to two control training conditions, one of which incorporates the typical measurement exercises from 2nd grade mathematics texts, and the other of which examines one aspect of the experimental condition, superimposition of units in the context of aligned ruler measurement. A third control condition, currently in progress, examines the other aspect of the experimental condition, comparing the measurement of objects that are initially misaligned with the start of the ruler and then aligned with the start of the ruler, but without the superimposition of discrete units. Preliminary results for this condition are considered in the Discussion section below.

Methods

Subjects

Fifty three second grade students, attending a private school in Chicago, were randomly assigned to each of the three conditions (range = 7.5 to 8.8 years; mean age = 8.0 years). There were 18 children in the Experimental Condition (8 boys, 10 girls); 17 children in Control Condition 1 (7 boys, 10 girls); and 18 children in Control Condition 2 (7 boys, 11 girls).

Procedure

All participants received a measurement pretest during the first session. During the second session, which occurred within a week of the first session, children received training and the immediate measurement post-test (Post-test 1). One week later, they received a delayed measurement post-test (Post-test 2). The pre-test and post-tests consisted of a multiple choice paper and pencil test in which children were given 16 measurement items (8 aligned and 8 unaligned). Each pretest and posttest item consisted of a picture of a ruler and a crayon that varied in length which appeared above the ruler, either at the "0" point, or in various misaligned positions. All items started and ended with a whole inch. Four numerical choices were provided below the ruler, one of which was the correct answer, one of which corresponded to the read off strategy, one of which corresponded to the counting hash marks strategy, and the other of which was a non-lure response.

In the second session, children received one of the training conditions (Experimental, Control 1, Control 2). In the Experimental Condition, the experimenter showed the child a stick that was placed just above the ruler at various points and asked the child how many units long the stick is. The experimenter then placed discrete one-inch long units (e.g. translucent red rectangular "chips") on top of the part of the ruler that was aligned with the stick, and again asked the child how many units long the stick is. After that, the experimenter moved the stick to the "0" point and the chips were placed on the part of the ruler that was aligned with the stick in its new position. This allowed the child to compare the measurements obtained in the unaligned and aligned positions and to see that they are the same. Each child was given 8 training trials, each involving a stick of a different length. In Control Condition 1, each child received 8 trials measuring sticks aligned with a ruler and 8 trials measuring sticks with discrete units. These activities were done separately, as is typically the case in school, rather than in a coordinated manner such as that used in the Experimental Condition. The two types of measurement were administered in a blocked, counterbalanced order. In Control Condition 2 the child received 8 trials in which each stick was presented aligned with the 0-point of the ruler. The child was asked how many units long the stick is, and after they answered, was asked to place the units on top of the ruler to see if they were correct. See Figure 2 for a depiction of the training conditions.

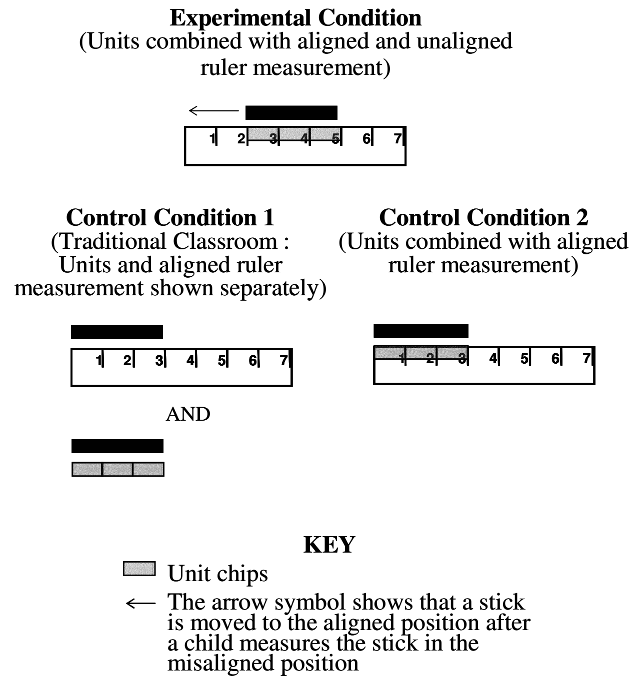

Figure 2: Depiction of the Experimental Condition, Control Condition 1 and Control Condition 2.

Results

The mean proportion of misaligned items correct (M = .28, SD = .38 at pre-test; M = .43, SD = .46 at post-test 1; M = .45, SD = .47 at post-test 2) was lower than that of aligned items correct (M = 1.00, SD = 0.00 at pre-test; M = 0.98, SD = .07 at post-test 1; M = 1.00, SD = 0.00 at post-test 2). Since performance on aligned items was virtually perfect at all three time points, we carried out a mixed-model ANOVA to examine effect of condition on children's performance on misaligned items. Condition (Experimental, Control 1, Control 2) was the between subjects factor and Time of Test (Pretest, Post-test 1, Post-test 2) was the within subjects factor. Results revealed a main effect of Time of Test (F (2,100) = 6.832, p < .005) and an interaction effect of Condition x Time of Test (F (4, 100) = 3.929, p <.01).

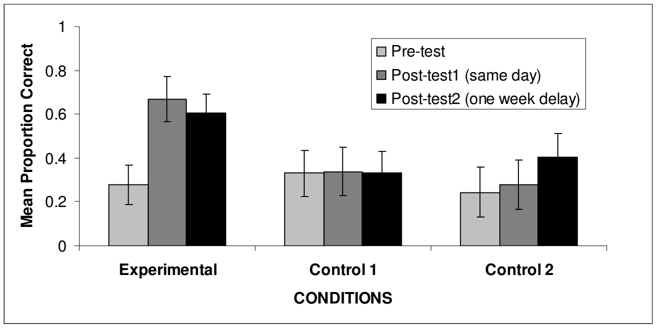

Figure 3: Mean proportion correct on unaligned problems Correct on pre-test, post-test1, post-test2 (one week later) for the Experimental Group, Control Group 1 and Control Group 2.

The main effect of Condition (p =.217) was not significant. Bonferroni tests revealed no differences between performance of the various groups at pre-test, *ps* >.9. The Experimental Group performed significantly better on unaligned items at post-test 1 $t(50) = 3.559$, $p < .05$ and post-test 2, $t(50) = 3.329$, $p < .05$ than at pretest (see Figure 3). There was no significant difference between performance of the Experimental Group at Post-tests 1 and 2, $p = .40$. In contrast, the two Control Groups did not show significant improvement between pretest and post-test 1 (Control Group 1: $p = .9$; Control Group 2: $p = .6$) or between pre-test and post-test 2 (Control Group 1: $p = 1$; Control Group 2: $p = .18$).

Our analyses also showed that the success of training in the Experimental Condition depended on the child's measurement strategy at pre-test. In the experimental condition, 7 out of 8 children who used hash marks strategy at the pre-test changed their strategy to counting units (inches) whereas none of children (0/5) who employed the read-off strategy in the experimental condition changed their strategy after training (Figure 4).

Discussion

Our findings show that young elementary school children benefit from instruction that makes comparisons between aligned and misaligned ruler measurement and coordinates the use of discrete units with unit markings on a ruler. Moreover, the results show that the kind of instruction that is typically used in classrooms, which makes use of separate activities involving aligned ruler measurement and measurement with discrete units is not effective in promoting the understanding of interval units. Indeed, even coordinating discrete units and aligned ruler measurement did not result in a significant improvement on misaligned ruler problems. Finally, our results show that children who count hash marks on misaligned ruler items are more prepared to learn about interval units than those who merely read off the rightmost number on the ruler on these items. Thus, hash mark counting, although an incorrect strategy, represents an important intermediate knowledge state.

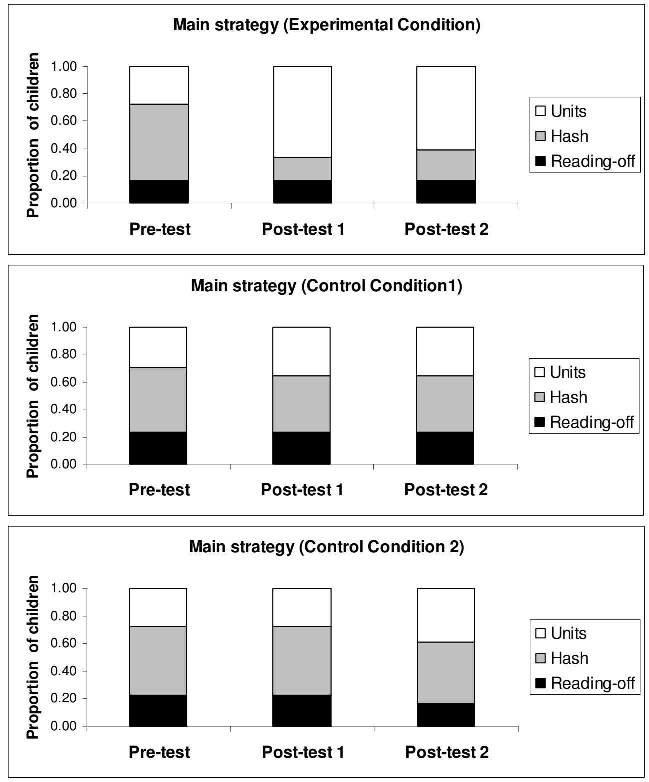

Figure 4. Proportion of children in Experimental Condition, Control Condition 1 and Control Condition 2 using each strategy at pretest, and proportion using each strategy at post-test 1.

Why is the experimental training condition successful while the control training conditions are not? We hypothesize that misaligned ruler measurement is helpful because it highlights the fact that the length of an item cannot be determined by merely reading off the rightmost number on a ruler. Further, combining the misaligned ruler measurement with discrete units shows children that counting the hash marks is not what measurement is about – rather it is about counting units. Finally, moving the object back to the 0-point shows that child that the answer obtained by reading off the rightmost number on the ruler on aligned problems derives from counting units. Thus, it moves children from the shallow understanding of *how to measure* with a ruler to the deeper understanding of *what ruler measurement is about and why it works.* In contrast to the Experimental Condition, Control Condition 1 mimics typical instructional strategies used in classrooms. Aligned ruler measurement shows the child that he/she can read off the rightmost number on the ruler and get the right answer. Measuring objects with discrete units such as paper clips has the potential to show the child that measurement is about counting interval units. However, it may not be easy for the child to link ruler measurement and measurement with discrete units because they are not perceptually aligned, an important feature of effective instruction (e.g., Loewenstein & Gentner, 2001). Control Condition 2 involves aligned ruler measurement and laying the units right on top of the ruler, which seems like it has the potential to be helpful because the units and the ruler are aligned. However, in this condition, the child can merely read off the rightmost number on the ruler to obtain the correct answer. Thus, showing the child that counting the discrete units yields the same answer may be viewed more like a parlor trick than about a deeply significant aspect of measurement. In an ongoing study, Control Condition 3, we are examining whether the comparison of misaligned and aligned ruler measurement used in the Experimental Condition would be effective without the use of superimposed discrete units. Our preliminary results suggest that this aspect of our Experimental Condition may be critical as those participants who have been assessed show significant gains at Post-test 1.

We suggest that the current findings are likely to have implications that extend beyond the case of ruler measurement. There are many situations in which we need to measure continuous extents. One example occurs when we apply distance scales such as 0.5 inch = 100 miles to assess the distance between locations shown on a map. The scale is not aligned with the distance we are interested in measuring and we not only need to apply it iteratively, but also to scale up the distance we measure on the map according to the scale factor. A sound understanding of units of measure also is critical to solving problems that commonly occur in mathematics and science, which often necessitate unit conversions. This is another kind of misalignment that may only be appreciated as critical if a strong understanding of measurement and measurement units is built from the earliest stages of learning.

Our future plans include extending this work to younger children and to classroom teaching environments. We also plan to study children's ruler measurement when object extents fall between inch markers. In this work we will ask whether children who succeed at unaligned whole unit measurement, perform with equal facility on aligned and unaligned problems involving fractional units. Researchers have posited that an understanding of the real numbers can be scaffolded by integrating numerical schemas with concrete non-numerical schemas such as the fullness of beakers (e.g., Moss & Case, 1999; Resnick & Singer, 1993, Sophian, 2007). We suggest that ruler measurement, if understood well, is a potentially fertile schema for improving young children's understanding of the real numbers and fractions because it provides an opportunity to link numerical values that are difficult to conceptualize to a concrete, ecologically valid, number line representation.

Acknowledgments

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